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Performance Analysis of Bang-Bang, Single PID, and Cascade PID Controllers for Cart Pendulum Stabilization

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Abstract

In midst of new development advance of controller in control theory studies, classical proportional-integral-derivative (PID) controller and it combinations are considered the most applicable and widely used controller. Certain industries prefer simple on/off controller or bang-bang controller for uncomplicated systems while some few others required complex analysis of system dynamics and advanced mathematics. PID controller is a simple yet effective controller that can do well on both cases. It is what makes it used in 95% of control loops process in industries. Conventional single PID controller increase performance of the system stability from basic bang-bang controller in low order system. While higher order system which mostly appears in real-life cases can be controlled with its variation cascade PID controller. This paper focus on how single and cascade PID controller implementation significantly improve performance of gravitational cart pendulum stabilization over simple bang-bang controller. The results show PID controller improve up to 10 times faster steady time and 5 times less overshoot over basic bang-bang controller. However the distance traveled for single PID controller is fairly high, to which cascade PID controller more suitable for by reduce distance traveled by half with slight performance trade off.

Keywords: Cascade Controller, Control Theory, Gravitational Cart Pendulum, PID Control, Stabilization

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I. INTRODUCTION

Automatic control system is one of branches in mathematics field which studies process and dynamics of both natural or artificial system to control it as desired. Control system studies holds important places nowadays as it helps improve the speed, accuracy, efficiency, and robustness of technological advancement. Some example of its application is a self driving car, the control is applied to manipulate the car speed, with several input such as the distance between the surrounding and the speedometer itself. Another example is in industrial process to control the production and manufacture certain parts or material to some detail spesifications [1]. As important as it is, the difficulty to understand the concept and applied it in real-life cases are very challenging. New discoveries of different type of controller with artificial intelligence and their complexity also makes studies harder for beginner to start. Many choose to learn through a simulation or a simpler system dynamics. Especially when the price of errors are very expensive or even fatal.

On/off control or bang-bang control is a simplest type of closed loop controller. One example is water heater that maintains desired tempereature by turning the applied water on or off. The controller only response by two states or binary value. Proportional-integral-derivative (PID) control is a type of close loop control with varying states which generate systems respons based on proportional, integral, and derivative term of error between reference value and controlled value [2]. PID control gives a simple feedback signal of the form “present-past-future”, which does not need an accurate mathematical models of the systems controlled. It eliminates a large error gaps through the proportional terms, eliminate steady-state error through integral terms, and prevents systems tendency to overshoot through derivative terms [3]. This concepts make PID controlled systems can be controlled through trial-error or by estimating the correct Kp, Ki, and Kd value based on basic understanding of the systems. For its simplicity, PID controller can be used in linear systems or systems with simple dynamics.

Problems would occure when conventional single PID controller applied in nonlinear and high disturbance systems such as actuator saturations, systems limit, accumulating and unpredictable noise. These problems can be solve using cascade control. Cascade controller is control technique with two closed loop control systems. First controller (master) gives an output based of first variable error, this output will be used as reference for second controller (slave) [4]. This controller scheme can be used to limit the response of the system. The design is intended to make more agile response, eliminates error of first controller, compensate for disturbance and limits of manipulated variables in systems, and eliminates nonlinearities [5]. Such advantage makes PID control more flexible and versatile in various systems.

This paper used pendulum as controlled system. Pendulum is one of the system used to study the dynamics and response against controller method. Pendulum dynamics also found in complex system such as segway, robotic manipulator, missile control, and even the dynamics bipedals walking. This paper focus on how to applied basic controller for pendulum such as bang-bang control, conventional single PID controller and cascade, dual-loop PID control in a way to show how certain change in control methods effects the stability of the system. The goal is not to implement a new controller method but rather to show the development of early simpler controller to more complex approach. Rise time, steady time, overshoot, and distance traveled will also be observe to show the performance improvement.

II. RESEARCH METHOD

1. *Pendulum system spesification*

The systems based on a cart pendulum with track length (l_c) of 37 cm long. At either side of the track is a 12 V DC motor and a rotary encoder. Both encoder and DC motor are connected by a belt and a pulley to move the pendulum cart back and forth. The resolutian of the track encoder is 400 ppr to measure the distance traveled by the cart. The cart itself carried the pendulum on 1000 ppr encoder to measure pendulum angle. The cart and encoder mass (m_c) is 225 gram while the pendulum mass (m_p) alone is 32 gram and 32 cm long (l_p). Cart pendulum dimention and parameter distance traveled (x) and pendulum angle (α) can be seen in Figure 1.

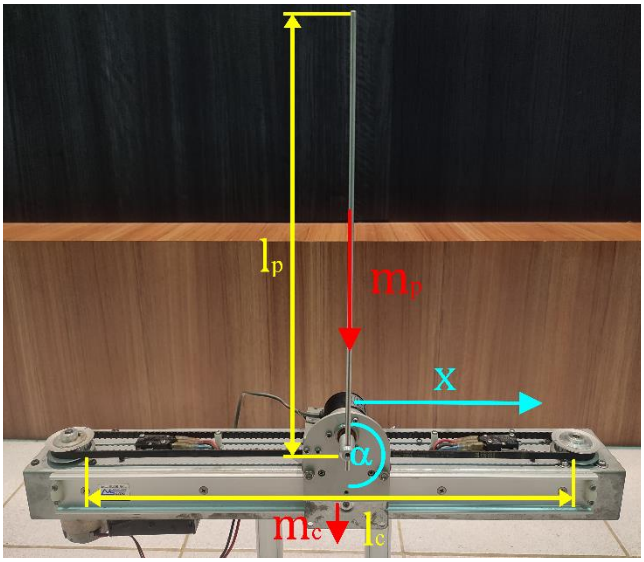

Fig. 1. Cart pendulum dimention

The controller used in the system is calculated within microcontroller ESP32. The microcontroller receive input of cart distance and pendulum angle, process it with controller algorithm, and produce output signal to control DC motor speed. The speed of the motor were regulated by driver motor H-Bridge BTS7960. Maximum speed rating of the motor without load is 3600 rpm on 12 V DC. Optical encoder measure pendulum angle and cart distance by counting digital signal every point passes in rotating motion. Pendulum components can be seen in Figure 2. By counting the point per rotation to degree in pendulum angle and distance traveled in cart movement, the resolution of angle and distance measurement will be acquired and shown in Equation (1) and (2).

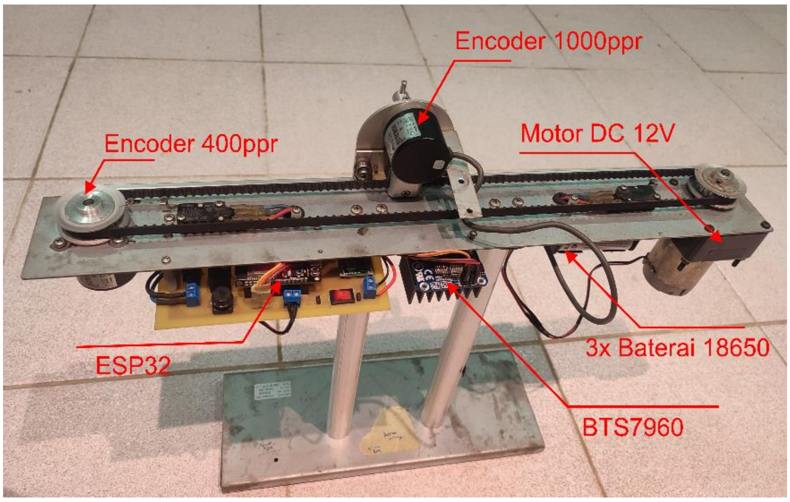

Fig. 2. Components used in cart pendulum

All the components used in this experiments works with digital input value and based on Yasuhiro Oyama [6] pendulum system with slight modification to adjust the system with common microcontroller. Since the encoder works by count rotation by point traveled in one single rotating movement, the metric value of the sensor needs to be calculated first.

$$1\ (\alpha) = \frac{1}{2000} \times 2\pi\, rad = 0.003142\, rad = 0.18° \tag{1}$$

Equation (1) shows how to calculate the angle resolution $(1(\alpha))$ in standar radian unit for pendulum angle measurement. One rotation in half-quad encoder mode for pendulum angle is counted as 2000, while one rotation in standar unit is define as $2\pi\ radian$. The result is a discrete and quantized value resolution for angle measurement.

$$1\ (x) = \frac{1}{800} \times \pi^2 = 0.012337\ cm = 0.000123\ m \tag{2}$$

Equation (2) shows how to calculate the distance resolution $(1(\theta))$ in standar metric unit for cart distance measurement. One rotation in half-quad encoder mode for cart distance is counted as 100, while the encoder diameter and circumference for cart system is define as π cm and π^2 cm. The result is a discrete and quantized value resolution for distance measurement.

2. Controller Design

PID controller is a continuous function value of integral and derivative of readings from sensor. However ESP32 microcontroller is a digital system. Besides, measurement resolution for angle in Equation (1) and distance measurement in Equation (2) show that the sensor input value is not an analog value but in discrete and quantized. An analog signal approach will not be sufficient to predict the system. To implement control algorithms into microcontroller, it is essential to transform math calculation into a discrete-time system [7]. The calculation for integral and derivative term in PID will using discrete time system as shown in Equation (3) and (4) below.

$$\int e(t) = \frac{Ts}{2}\left[\left(e(t) + 2e(t-1) + e(t-2)\right)\right] + \int e(t-1) \tag{3}$$

$$\frac{de(t)}{dt} = \frac{Ts}{2}\left(e(t) + e(t-1)\right) \tag{4}$$

Where e(t) is error or difference between angle θ (in radian) or distance x (in meters) reference value and measured value. Ts or sampling time is equal to 0.001 s.

The pendulum system is controlled by three types of controller, bang-bang control, conventional single PID controller and cascade, dual-loop PID control. The system will be given disturbance input to trigger controller response to stabilize pendulum in regular downward position. First method, bang-bang controller works by moving cart back and forth in constant speed if pendulum angle reach certain value. Second method, single PID controller stabilize pendulum with different motor speed depends on system error and Kp, Ki, and Kd Value [8]. Third method, cascade PID controller use the output of first PID controller as reference point for new output to motor speed [5] [9]. Block diagram of each controller is shown at Figure 3, 4, and 5 respectively.

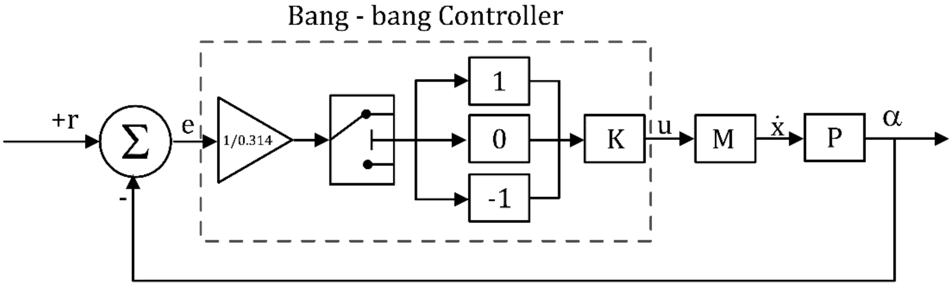

Fig. 3. Block diagram of bang-bang control

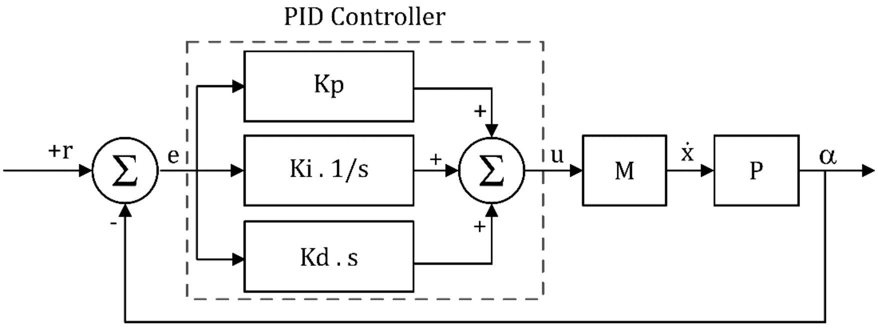

Fig. 4. Block diagram of Single PID Controller

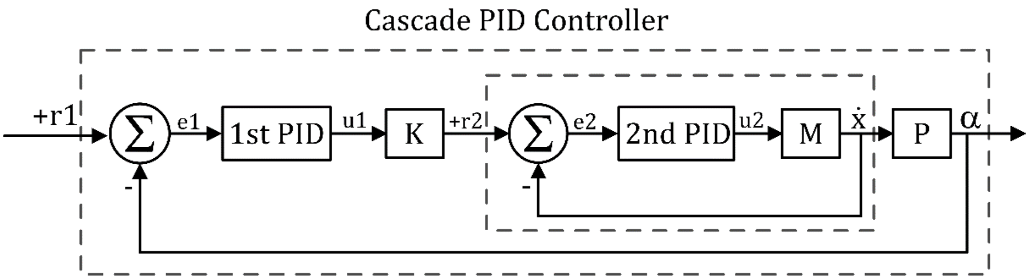

Fig. 5. Block diagram of Cascade PID Controller

System parameters will be measured for comparison of each controller. Such parameter is rise time, settling time, overshoot, and distance traveled. Since gravitational pendulum always point downwards, error steady state calculation wouldnt be necessary. Data will be measured by microcontroller ESP32 using built-in timer and serial communication with computer device. The system will be given deviation of 30°, 60°, and 90° from stable downward position before each controller start.

III. RESULTS AND DISCUSSION

1. Bang – bang Controller

Measurement for bang-bang control analysis was taken using three different initial angle, each measurement repeated for 5 times. Here table (I) below shows average system parameter after controlled by bang-bang controller.

Table I. Parameter systems controller by Bang-bang Controller

Initial Angle	**Test**	**Rise Time (s)**	**Steady Time(s)**	**Overshoot**	**Distance (cm)**
30°	1	0.277	7.432	13.68°	3.9
	2	0.242	10.043	18.18°	3.8
	3	0.211	9.592	18.36°	2.8
	4	0.335	7.470	13.50°	3.4
	5	0.211	10.047	17.28°	3.7
60°	1	0.308	9.702	25.20°	1.9
	2	0.345	9.766	24.84°	3.6
	3	0.318	9.743	25.74°	1.9
	4	0.299	9.735	27.36°	1.1
	5	0.294	9.716	25.92°	1.5
90°	1	0.394	10.274	52.02°	6.5
	2	0.380	10.689	52.20°	7.4
	3	0.363	10.689	51.48°	4.8
	4	0.360	10.243	47.70°	5.4
	5	0.378	10.716	52.74°	6.1

The table shows problems before implementation of PID control. Bang-bang control or on/off control only move the system back and forth at constant speed every times pendulum angle pass through upper or below limit. System response speed did not adapt to error value which makes system gives response at the same speed regardless of initial condition. System parameter also shows high overshoot up to half of initial angle due to inadaptability to error value. These problems cause system took long time to reach stable position.

2. Single Conventional PID Control

Single conventional PID control was design to solve problem at basic on/off controller. It gives quick response to high error differences, while maintaining speed near setpoint to avoid overshoot. Table (II) below shows measurement result for system parameter controlled by single PID controller.

Table II. Parameter systems controller by Single Conventional PID Control

Initial Angle	**Test**	**Rise Time (s)**	**Steady Time(s)**	**Overshoot**	**Distance (cm)**
30°	1	0.130	0.382	5.76°	8.9
	2	0.133	0.470	6.84°	9.7
	3	0.130	0.395	6.66°	9.1
	4	0.127	0.424	6.30°	8.4
	5	0.129	0.401	6.66°	8.6
60°	1	0.200	0.528	8.46°	20.2
	2	0.186	0.416	11.70°	20.5
	3	0.195	0.517	8.82°	19.8
	4	0.195	0.543	9.54°	18.6
	5	0.190	0.527	9.00°	19.1
90°	1	0.319	0.726	17.10°	34.7
	2	0.304	0.698	16.74°	33.4
	3	0.301	0.696	16.38°	32.8
	4	0.339	0.751	17.82°	36.5
	5	0.318	0.618	13.32°	36.3

Parameter system showed big improvement in implementation of PID controller. System response at higher speed and reach setpoint in 0.13 seconds. System steady time shows significant differences to reach steady state with duration less than a second at 0.38-0.62 seconds. Maximum overshoot also didnt get as big as bang-bang controller at 5.76° - 13.32°. These three parameters shows efectivity of PID control system to stablize system.

PID control widely used in industrial application due to its effectiveness and simplicity. However systems in real-life application has limitation and non-linear hence why there is a lot of difference approach in PID implementation for difference system. In this gravitational pendulum system, when presented with highr error controller gives response by moving pendulum cart at higher speed. The distance measured by PID control as shown by table (II) significantly higher than bang-bang control. Consequently due to track distance limitation, difference approach needed to avoid collision in the system.

3. Cascade PID Control

First PID control in the system designed to gives response for error in pendulum angle similar to single PID controller. In cascade PID Controller, the output (u_1) is used as setpoint for second PID controller. A constant K used as input ratio between angle and distance error in total system. This design purpose is to gives quick response similar to single PID controller, but also measure distance as system limitation. Cascade PID control for this system gives result as shown in table (III) below.

Table III. Parameter systems controller by Cascade PID Control

Initial Angle	**Test**	**Rise Time (s)**	**Steady Time(s)**	**Overshoot**	**Distance (cm)**
30°	1	0.161	0.643	5.76°	6.5
	2	0.157	0.632	5.76°	6.4
	3	0.168	0.645	5.76°	6.7
	4	0.132	0.524	5.76°	5.0
	5	0.176	0.680	6.12°	6.8
60°	1	0.257	0.897	11.88°	14.0
	2	0.264	0.907	12.42°	14.2
	3	0.255	0.903	12.60°	14.1
	4	0.251	0.899	12.06°	13.7
	5	0.252	0.884	11.16°	13.0
90°	1	0.357	1.650	27.36°	19.0
	2	0.358	1.640	28.44°	18.7
	3	0.343	1.620	26.64°	18.8
	4	0.343	1.622	26.46°	18.6
	5	0.364	1.646	28.98°	19.0

Table (III) shows cascade PID systems gives quick rise time similar to single PID controller while limit distance traveled by cart to avoid collision. However system steady time is reduced by a few milisecond, and overshoot at higher error also increased significantly.

Comparison between system parameter for each controller shows importance of PID control to stabilize system over bang-bang control. Figure 6 show advantage of PID controller over bang bang controller with quickest rise time. In these parameter, single PID controller proven its advantage over cascade PID controller.

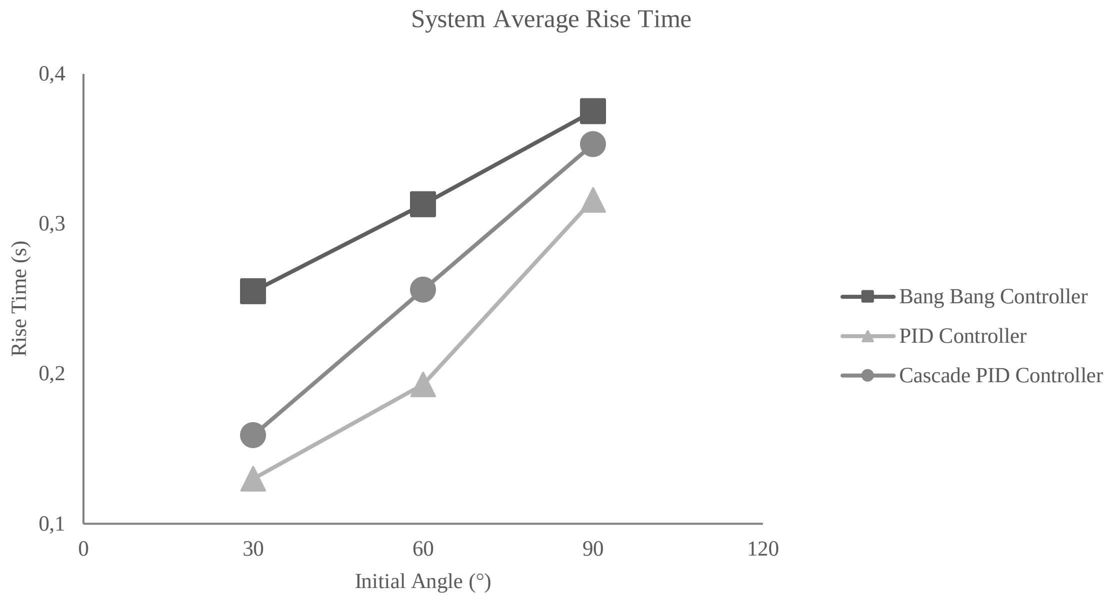

Fig. 6. System average rise time for each controller

System steady time and overshoot behave similar to rise time, both PID controller method shows significance performance over bang bang controller though single PID controllers are slightly better than cascade PID controller. However in smaller 30° and 60° initial angle, both PID controllers shows little to no differences. This could happen as smaller degree deviation caused little movement by cart which makes cascade PID controller works the same as single PID controller.

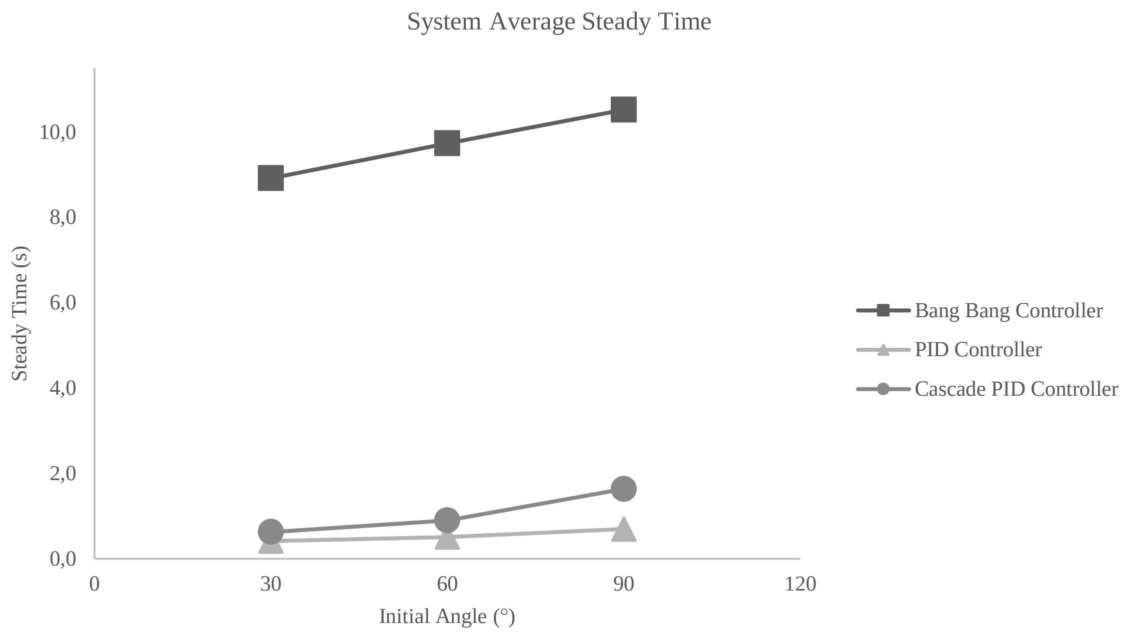

Fig. 7. System average steady time for each controller

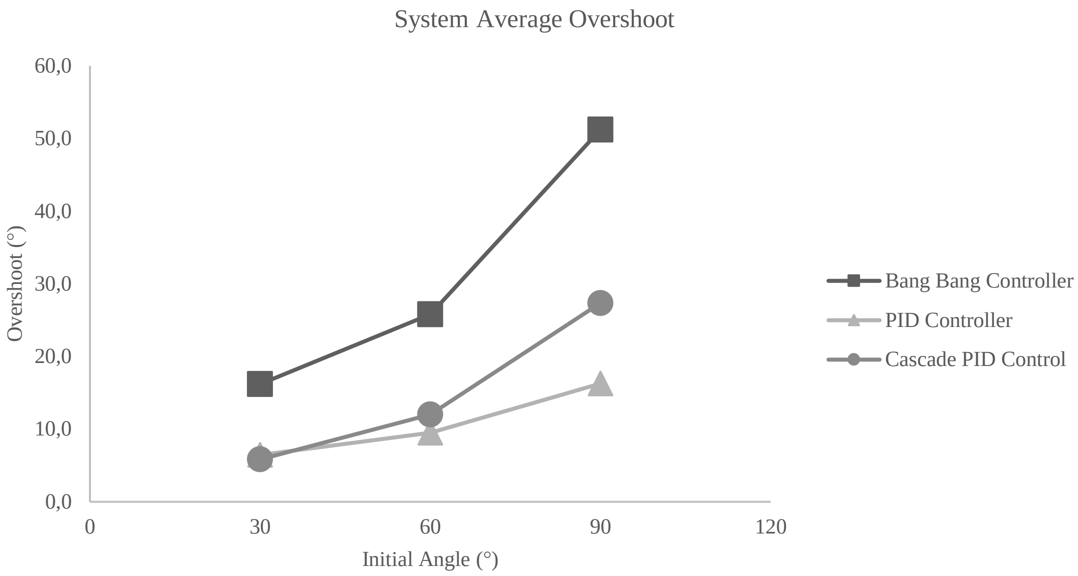

Fig. 8. System average overshoot for each controller

In real life application, control system often met with limitation by hardware or actuator saturation. In case of cart pendulum, the system are limited by its track length which make cascade PID control are preferrable. Figure 9 shows differences between maximum distance movement by each controller.

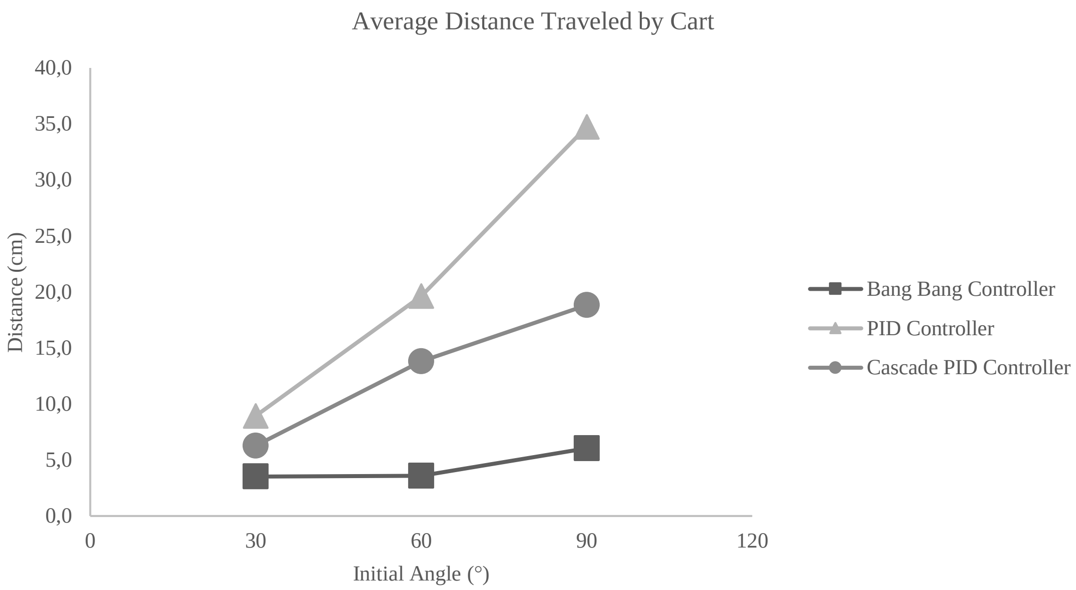

Fig. 9. Average distance traveled by cart for each controller

IV. CONCLUSION

PID control and its variations are most widely used in industrial implementation. It is very effective yet simple to control system dynamics in general. The result of this paper shows that PID control is very effective to stabilize gravitational cart pendulum system with 10 times faster steady time, 2 times faster rise time and 5 times reduced overshoot over basic bang-bang controller. Though the distance traveled for PID controller is quite far up to 36 cm which shows real-life application of PID controller requires important understanding of system limitation to which cascade PID controller is perfect to applied. Cascade PID controller consider the cart distance from track center point to its control signal. The result with cascade PID controller shows maximum distance of 19 cm traveled by the cart, 2 times less over single PID controller. But it also comes with slight increase in steady time and overshoot.

PID control is overall much better to applied over bang-bang control. However it implementation needs basic understanding of system controlled. Single PID control works perfectly for absolute performance of one system parameter regardless of other variable, while cascade PID control suitable to stabilize system with high enough performance and certain limitation.

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